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Review of Reliability Techniques

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REVIEW OF RELIABILITY TECHNIQUES

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Eugene R. Doherty

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INTRODUCTION

The reliability problem

In the development of any product to perform a specific function the first concern of the engineer is to design for satisfactory operation. Engineers originally approached the reliability problem by using excessive safety factors to be assured the structure or material would withstand the calculated loads and stresses. The engineer also learned from operating or testing the equipment until failures occurred and then redesigning as mistakes became apparent. These methods were time consuming and often resulted in bulky over designed products. These approaches became impractical with the advent of new technological advancements. The accelerated industrial development of aircraft, missiles, and modern electronics coupled with a need for a drastic reduction in weight and size magnified the problem.

As products became more complex the problem of building a reliable product was intensified. An appreciation for the increase in complexity can be gained from considering that in a period of fifteen years the requirements for electronic tubes on a U. S. Navy destroyer changed from sixty to thirty-six hundred (14). During World War II new equipment was developed that had to be operational for extended period of time if the military mission was to be accomplished. The addition of a time requirement added to the already difficult problem caused by the increasing complexity of equipment. It soon became obvious that new techniques had to be developed that would assist the manufacturer in designing a reliable product.

Historical background

It has been observed that the reliability problem was always present during the development of a product, and with the transition to complex electronics a new approach was required. It is impossible to determine the exact start in establishing these methods and techniques which conceived the new discipline, "reliability". However, sone of the first published literature appears in the early part of 1950 with a specific interest in electronic equipment. In 1952 the Advisory Group on Reliability of Electronic Equipment (AGREE) was established. This group was comprised of representatives from both the government and industry with the responsibility of stimulating interest in reliability matters and recommending measures which would result in more reliable electronic equipment. This group (16) published the findings of nine task groups that had been established to study and prepare recommendations on specific areas of the reliability problem. Generalized assignments for each task group are as follows:

Task group 1 - Establish minimum acceptable requirements, apportioning reliability responsibility to portions of a system.

Task group 2 - Determine methods for assessing reliability and measuring mean-time-between-failure in prototype models, assuming the exponential distribution.

Task group 3 - Provide test plans for determining compliance with mean-time-between-failure level and longevity, assuming the exponential distribution.

Task group 4 - Establish a development procedure, assuming that a quantitative level of reliability is specified, to insure the attainment of inherent reliability.

Task group 5 - Develop criteria and methods for specifying the reliability of electronic component parts and tubes in terms of failure rate as a function of time and environment.

Task group 6 - Contractual aspects of procurement of reliability.

Task group 7 - Packaging effects on reliability.

Task group 8 - Storage effects on reliability.

Task group 9 - Maintenance and maintainability relative to reliability.

Results of these early studies stimulated changes in military specifications establishing reliability requirements. This in turn promoted reliability in industry and started research in statistical methods and techniques applicable to the reliability problem.

Some of the pioneers in individual literature on reliability would include Robert Lusser, B. Epstein, M. Sobel, and D. J. Davis (11, 5, and 3).

Organization

Placement of the reliability function within the organizational structure is a very controversial subject in reliability today. Reliability responsibilities have been delegated and found in a variety of places. Some of these appear to be more political rather than practical. In certain instances quality control is charged with the entire responsibility for reliability. This is because frequently the emphasis is on data reporting systems, vendor control, and statistical services. In other cases the reliability function is found completely within engineering where it is associated with design and development. A reliability committee consisting of members from many departments has been used to advise higher management on actions which should be taken. None of these seem to be completely satisfactory. Quality control cannot supervise reliability activities in engineering. Equally, engineering cannot implement a reliability program in manufacturing. There is further difficulty in having the reliability responsibility in a single department. In such cases it is quite possible that the reliability function will be used to serve a selfish objective of departmental growth. That is to say that the over-all program would be neglected when confined to one department.

> Let us first list those activities for which a Reliability Group either has direct responsibility or participates in an advisory or secondary capacity. These functions are:
>
> 1. Reliability evaluation
> 2. Reliability apportionment
> 3. Design review
> 4. Design control
> 5. Specification, material, and processing review
> 6. Vendor control
> 7. Test planning, operation, and analysis
> 8. Reliability knowledge
> 9. Reliability and failure reporting systems
> 10. Mathematical and statistical services for reliability problems
> 11. Reliability education
> 12. Internal coordination of reliability activities. (10, p. 16)

It is apparent reliability responsibilities extend beyond engineering only, manufacturing only, or purchasing only, but rather encompasses all of them. The organizational structure must be such that the reliability activities in all areas can be coordinated so that everyone works toward the objective of producing a reliable product.

The manner in which an over-all reliability program is organized depends on the size of the company, the type of product, and the production quantity. However, to be effective the function must be recognized and its manager must have the power to make decisions and to enforce them in any area where the need arises. Clearly no reliability program can succeed unless the top management is convinced of the need for the program. If management were sufficiently aware of the importance of this group it would no doubt report to the chief executive along with the other major functions of the company. Since this will be some years in coming, the reliability group should at least report to the first level of management as a staff organization.

One of the most important activities of the reliability group is that of reliability education. Personnel in management, engineering, manufacturing, and purchasing must be cognizant of reliability. All engineering personnel should be offered an education in reliability principles, theory, and techniques. When this brings sufficient enlightenment reliability could become a product characteristic as proposed by Hadley and Bruce (7).

> Over the long range, the need for a special group called "reliability" will vanish as standard departmental

> functions become able to absorb their responsibilities. This will become possible as the technical levels of all functions improve until finally all engineers and specialists in purchasing, quality control, and manufacturing are more knowledgeable in electronics methodology. At that future time, organization charts will look much as they do today, and reliability will be another characteristic specified by engineering and measured by quality control. (7, p. 51)

Data requirements

Very important procedures of any reliability program are the collecting, processing and reporting of data. It requires considerable planning to implement a system that will provide adequate data in a form suitable for reliability analyses and predictions. The value of timeliness and precision of the basic data must not be underestimated, since estimates derived from these data determine the success or failure of the reliability program.

A complete failure data collection and processing system is best designed to the so-called closed loop concept of discovery, analysis, and corrective action. This concept provides the flow of information required to incorporate design changes early in the research and development program and minimizes changes in the production program. It also provides an early accumulation of historical data which can be intergrated with quality control and quality assurance test results in defining operating characteristics of the final product.

Methods, procedures, and environmental conditions applicable to these data, which are obtained from past experience, should be clearly specified in order to utilize the results in any later reliability evaluations. If statistical design of experiments have been utilized in the planning with adequate control, then the precision of the reliability prediction is definitely increased. It is possible the effect of one operating characteristic is sufficient to define the reliability of a component part.

There is a need for a method of exchanging reliability data within industry to better utilize established results. The present industrial Data Exchange Program (I.D.E.P.) does not completely satisfy this need, but could do so if industry were more cooperative. Probably the greatest single problem in such an exchange program is the standardization of test procedures, test requirements, and methods of data reduction. However, if such a program could be initiated it would contribute highly to the statistical confidence associated with reliability predictions.

THE RELIABILITY CONCEPT

Reliability definition

Authorities may vary slightly in defining reliability, but the classical definition could be the one proposed by AGREE: "Reliability is the probability of performing without failure a specified function under given conditions for a specified period of time." (16, p. 118) This definition stresses four elements specifically: Probability, performing without failure, given conditions, and time. An appreciation of each is necessary in understanding the concept of reliability, Probability, the first element of the reliability definition, is a quantitative expression such as a fraction or a percent which signifies the chance of an event occurring. Without failure, the second element of the reliability definition, infers success. Therefore, criteria must be established that defines successful operation. Conditions, the third element of the reliability definition, refers to environmental factors such as temperature, humidity, shock and stress. These factors must be considered with respect to their effect on performance when establishing success criteria. The last element of the reliability definition, time, provides interval measurements of operating life.

It can be observed from the above that operating conditions and operating time are defining characteristics of successful operation. Therefore, reliability can technically be defined in its simplest form as the probability of success; conversely, unreliability is the probability of failure.

Probability

Since probability is the prime element of reliability and the foundation upon which statistical theory and methods are based, it is necessary to have an understanding of basic probability.

Consider a random series of observations or experiments under uniform conditions which are observed with respect to the occurrence or nonoccurrence of an event A . The event A will occur a certain number of times, say f in the total number of n observation. Call the ratio f/n the relative frequency of A in the n trials. Generally this ratio will approach some constant for a large n . Assume it is possible to determine a number P such that, as n becomes very large, the ratio f/n will be approximately equal to P . This number P is referred to as the probability of the event A . The probability of the occurrence of the event is designated by $P(A)$ and the nonoccurrence by $P(\overline{A})$. Consequently, probability is a number between zero and one as $P(A)$ plus the $P(\overline{A})$ must equal unity.

Probabilities of two or more events can be combined by specific rules governing event operations. First, consider events A and B . Then,

$$P(A \text{ or } B) = P(A) + P(B) - P(AB)$$

Where $P(AB)$ is the probability of the joint or simultaneous occurrence of A and B . If A and B are mutually exclusive (that is, the occurence of one precludes the occurrence of the other), then

$$P(A \text{ or } B) = P(A) + P(B) .$$

This may be generalized for the probability that one of n mutually exclusive events occurs as the sum of the probabilities that each individual event occurs:

$$P(A_1 \text{ or } A_2 \text{ or} \ldots A_n) = \sum_{\lambda=1}^{n} P(A_i) .$$

Secondly, consider the joint or simultaneous occurrence of events A and B If the events A and B are statistically independent, that is, the occurrence of A does not depend on the occurrence of B and vice versa, then

$$P(AB) = P(A) \cdot P(B) \cdot$$

This may be generalized for the concurrent probability of n statistically independent events as the product of the individual event probabilities:

$$P(A_1 A_2 \ldots A_n) = \prod_{i=1}^{n} P(A_i)$$

These two probability rules are applied in reliability and referred to as the reliability addition law and the reliability product law respectively.

Series systems

Reliabilities for components are derived from test results which yield probabilistic failure rates where $P_i = R_i$ or the probability of success of the i^{th} component is the component reliability.

The system aspect of reliability concerns the functional relationship between the reliability of individual components and the reliability of the system. If a number of components of a system are connected in such a way that the failure of any one component causes a failure of the system, then these components are said to be functionally in series. The reliability of a system of series components which are statistically independent is the product of the reliabilities of the components.

For n components in series the system reliability is given by

$$R_s = R_1 \cdot R_2 \cdot R_3 \cdots R_n = \prod_{i=1}^{n} R_i$$

Where R_i is the reliability of the i^{th} component in series in the system.

Parallel systems

A system which has a component paralleled functionally such that a single failure doesn't result in a system failure is generally referred to in reliability as redundancy. When there are multiple redundant components in a system it is possible to have one or more component failures and have the system operate successfully.

If a simple system is defined by two parallel switches, say A and B, and the probability of successful closure is 0.995 for each, then what is the reliability of the system? This can be determined by considering that successful operation of the system can occur in three ways:

1. A is closed and B is open or $P(A)\,[1 - P(B)]$
2. B is closed and A is open or $P(B)\,[1 - P(A)]$
3. A is closed and B is closed or $P(A)\,P(B)$.

These three events 1, 2 and 3 are mutually exclusive and therefore can be expressed as the sum of the probabilities that each individual event occurs:

$$P\,(1 \text{ or } 2 \text{ or } 3) = P(A)\,[1 - P(B)] + P(B)\,[1 - P(A)] + [P(A)\,P(B)$$

$$= P(A) - P(A)\,P(B) + P(B) - P(A)\,P(B) + P(A)\,P(B)$$

$$= P(A) + P(B) - P(A)\,P(B)\,.$$

In reliability terminology with R_p defined as parallel reliability this is

$$R_p = R_a + R_b - R_a\,R_b$$

$$= 0.995 + 0.995 - (0.995)^2$$

$$= 0.999975.$$

Note that the parallel system reliability is higher than the individual switch reliabilities.

The reliability of two parallel switches can also be calculated in another way. The system can fail in only one way which is when A and B both fail to close. The probability of this happening is

$$P(\overline{A})\,P(\overline{B}) = (0.005)\,(0.005) = 0.000025$$

This can be defined by Q_p which is the parallel unreliability which can be subtracted from unity to obtain the reliability. For n components in parallel the system unreliability is

$$Q_p = Q_1 \cdot Q_2 \cdot Q_3 \cdots Q_n = \prod_{i=1}^{n} Q_i$$

And the reliability is

$$R_p = 1 - Q_p\,.$$

Reliability Failures

Reliability authorities distinguish three characteristic types of failures which occur in the life of equipment. These include early, chance or random, and wearout failures which when plotted as a smoothed failure rate versus time curve result in the life characteristic curve of Figure 1.

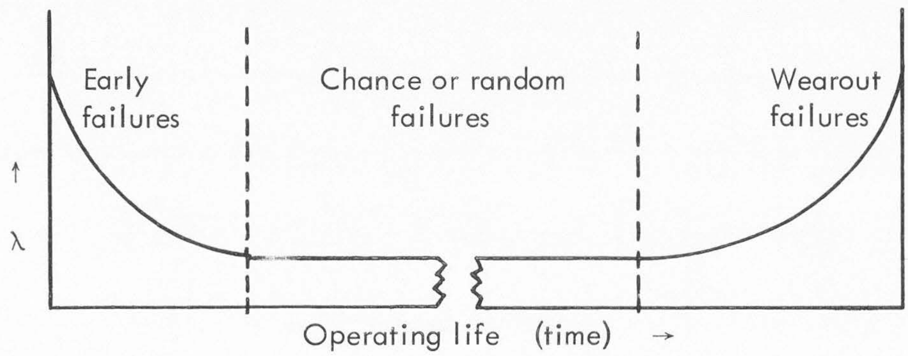

Figure 1. Failure rate versus time

This curve is sometimes referred to as the reliability bathtub curve because of its obvious cross-sectional resemblance. The early portion of this curve represents a common characteristic of most complex devices to contain at manufacture a number of marginal and short life parts. The chance or random portion is smoothed out into a constant failure rate. When wearout is reached the failure rate increases rapidly.

Reliability theory and practice differentiates between early, chance or random, and wearout failures mainly because specific failure distributions represent each of the three types of failures.

Statistics provide exact formulas for the frequency of occurrence of events following various kinds of statistical distributions from which the probability of occurrence or reliability can be derived. The events of concern in reliability are those that occur in the time domain. As an example, consider wearout failures which usually cluster around a mean wearout time. Once their statistical distribution is known, the probability of wearout failure occurrence at any operating time can be calculated. Similar considerations apply to early failures and to chance failures. However, early and chance failures do not cluster around any mean life but occur at random intervals. Therefore, they belong in the category of random events with their own characteristic distribution.

Failures and failure rates are generally associated in the reliability literature to components because large complex systems seldom can be sufficiently tested to establish system failure rates. Therefore, it is necessary to utilize component failure rates in estimating system reliability. These can be combined by the basic probability rules, but it requires operational knowledge of the component structure within the system.

Reliability measurements

Reliability measurements are basically time measurements from which the probability of an event in the time domain is estimated. The estimation falls into the category of statistical data evaluation.

Estimates of reliability from data in the form of success or failure, giving no information other than the fact the event occurred or did not occur, is the ratio of successes to total trials. This estimate can be related to operating time by defining success for a specific time. The technique used is based on the probability definition of reliability for which the estimated reliability for any operating time t is

$$R(t) = \frac{S(t)}{N}$$

Where N is the initial number of items subjected to a test and $S(t)$ is the number of items which did not fail up to time t. If N is the total population of some component then the ratio is the true probability which is defined for attribute data as the limit of the ratio of favorable events to the total number of trials as the number of trials approaches infinity.

Measurements of variable data, which is data possessing additional specific information such as successful operation for some measured length of time, are mean time to failure and mean time between failures. The term MTTF, mean time to failure, is used in the case of simple components which are replaced as failures occur. The operating times can be measured exactly, and the mean time to failure is then the sum of the t_i times of all n components divided by n, the number of all components. The term MTBF, mean time between failures, is simply the total operating time divided by the total number of failures. If operating life of a component is defined by cycles of operation, then mean cycles between failures is the measurement of reliability.

MATHEMATICS OF RELIABILITY

Statistical probability

The importance of the concept of statistical probability cannot be over-emphasized. One consideration of statistics is to predict from a number of observations, which have an element of uncertainty, the probability of observations falling in a given interval or the probability of any event out of a given set of events. An experiment may be repeated many times under similar conditions and have an uncontrollable variation which is random so that the observations are individually unpredictable. However, the observation will fall into certain intervals where the relative frequencies or percentage frequencies are quite stable. Then the relative frequency of the occurrence of an event can be used as an approximation of the probability of the event.

A population distribution of a random variable X shows the relative frequencies of occurrence of all possible values of X. Sometimes the distribution is represented by a curve $f(X)$, which is approximated by a relative frequency histogram for a large sample from the population. There are two types of random variables to be considered with respect to representative distributions. Mood and Graybill (13) define random variables as discrete and continuous.

Definition 3.1. X will be called a (one dimensional) discrete random variable if it is a random variable that assumes only a finite or denumerable number of values on the X axis. Suppose that X assumes only the values

$X_1, X_2, \cdots, X_n,$ ... and with probabilities

$f(X_1), f(X_2), \cdots, f(X_n), \ldots$ and suppose that A

is any subset of the points $X_1, \cdots, X_n,$ ... .

Then the probability of the event A (The probability that X is in A), written $P(A)$, is defined as

$$P(A) = \sum_{a} f(X)$$

Where $\sum_{a} f(X)$ means sum $f(X)$ over those values X_i that are in A .

Definition 4.1. The random variable X will be called a (one-dimensional) continuous random variable if there exists a function f such that $f(X) \geq 0$ for all X in the interval $-\infty < x < \infty$ and such that for any event A

$$P(A) = P(X \text{ is in } A) = \int_{a} f(X)\, d$$

$f(X)$ is called the density of X , and we shall sometimes say "X" is distributed as $f(X)$ or "$f(X)$ is the distribution of X ." (13, p. 53 and 80)

Therefore, a density function $f(X)$ of a random variable X gives the probability that any specified value of X will occur. Any function can be the density of a discrete random variable if it satisfies.

$$f(X_i) \geq 0 \qquad i = 1, 2, \cdots$$

$$\sum f(X_i) = 1$$

or of a continuous random variable if it satisfies

$$f(X_i) \geq 0 \quad -\infty < x < \infty$$

$$\int_{\infty}^{\infty} f(X)\, dx = 1 .$$

Since the density $f(X)$ is a probability the distribution of $f(X)$ is frequently referred to as a probability distribution. In relation to random variables there are two types of probability distributions called discrete and continuous.

Discrete distributions

A function, $f(X)$, which generates the probabilities that the chance variable will assume specific discrete values is a probability function. The total probability up to and including X, is the function, $F(X)$, which is called cumulative distribution function and is defined as

$$F(X) = \sum_{X' \leq X} f(X')$$

for all values of X.

The binomial probability function is probably the most frequently applied discrete probability function. It is a probability function defined by two parameters, p and n, and is associated with repeated independent trials of the same event. The probability of success on any trial is generally defined as p and the probability of failure as q or $(1-p)$. Then the probability of X the number of successes in n trials is

$$f(X) = \binom{n}{x} p^x q^{n-x}$$

$$\binom{n}{x} = \frac{n!}{x!\,(n-x)!} \qquad X = 0, 1, 2, \cdots, n ,$$

The function actually represents a family of distributions, with a specific distribution given when p and n are assigned specific values.

A random variable x is said to be distributed as a poisson if the density is

$$f(X) = \frac{\lambda^x e^{-\lambda}}{x!}$$
$$x = 0, 1, 2, \cdots$$

with an infinite range for x . This discrete distribution is frequently used as an approximation to the binomial when n is large and p is very small. The parameter λ is the average number of times the event occurs.

<u>Continuous distributions</u>

If a random variable can assume all values within an interval, the cumulative distribution function is

$$F(X) = \int f(X)\, dx$$

and is defined as a continuous distribution. Three continuous distributions frequently utilized in reliability are the normal, exponential, and Weibull.

The normal or gaussian distribution is a very important distribution as it is the foundation of a large number of the techniques used in applied statistics. The density function associated with this distribution is

$$f(X) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(X-\mu)^2/2\sigma^2}$$

$$-\infty < x < \infty$$

where X is the random variable and μ and σ are parameters referred to as the mean and standard deviation respectively.

The exponential distribution is characterized by a constant failure rate λ which fully describes the given distribution. The exponential density function is

$$f(t) = \lambda e^{-\lambda t}$$

where t is the random variable and λ is the single defining parameter. The obvious mathematical simplicity of the exponential distribution as represented by the single parameter, λ, has resulted in its extensive use in the field of reliability.

The unique feature of the three-parameter Weibull distribution is that the failure rate may be increasing, decreasing, or constant. It has a failure density which, as a function of time (t), can be expressed as

$$f(t) = \frac{\beta(t-c)^{\beta-1}}{\alpha} \quad e - \frac{(t-c)^{\beta}}{\alpha}$$

$$t > 0 \qquad \alpha > 0 \qquad \beta > 0 \qquad C \geq 0$$

where α is the scale parameter, β is the shape parameter, and C

is the location parameter. The shape parameter determines the general appearance of the distribution. If the failure rate is increasing $\beta > 1$ if the rate is decreasing $\beta < 1$, and if the rate is constant $\beta = 1$. When $\beta = 1$ this distribution is equivalent to the exponential distribution.

BINOMIAL DISTRIBUTION

Probability function

A frequently encountered type of discrete distribution is the binomial distribution which generates the probabilities of a random variable over a specified sample space. The binomial distribution applies to problems where the events occur independently such as flights of a missile. Further, the random variable is characterized by its limitation of being classified for two possible outcomes, which could be success or failure, and by the associated probabilities which are constant from trial to trial.

The binomial probability function was previously defined as

$$f(X) = \binom{n}{x} p^{x} q^{n-x}$$

$$x = 1, 2, \cdots, n \qquad q = 1 - p$$

which provides an estimate of the probability of exactly x successes in the n trials dependent upon the associated constant probability p. The cumulative distribution of these probabilities is expressed as

$$F(X) = \sum f(X_i) ,$$

and the sum of all the probabilities within the sample space will equal unity. A typical binomial distribution is shown graphically in Figure 2 for $n = 10$

and $p = 0.25$ which, as an example, is interpreted that the probability of exactly 2 successes is approximately 0.28

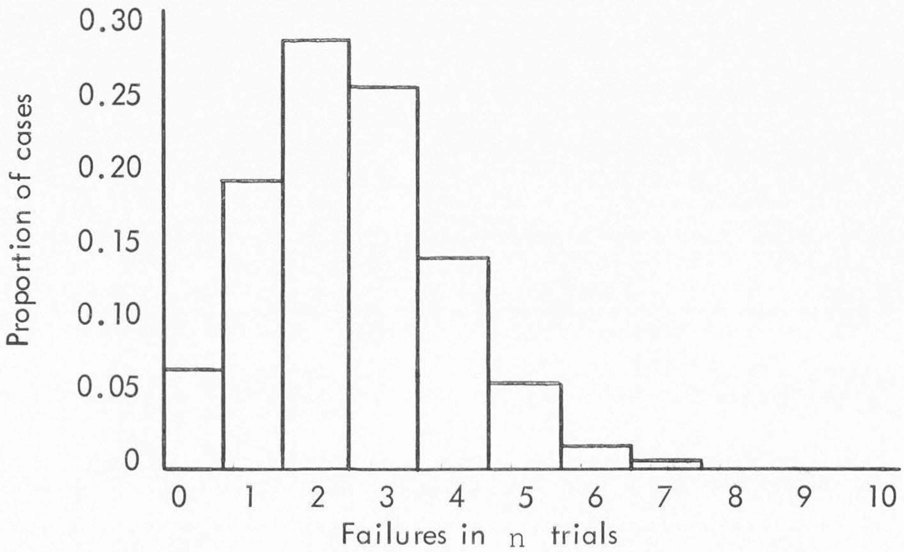

Figure 2. Binomial distribution $n = 10$ and $p = .25$

Reliability application

In reliability testing some systems operate in a discrete manner, that is when each test is complete the classification of the result is favorable or unfavorable. These systems may include such devices as atomic warheads, one-shot components, shells, electric fuses, etc. A point estimate of reliability is obtained from the ratio of favorable outcomes to the total number of tests, if each test has an equal chance of resulting in a favorable outcome. This reliability is an approximation or reliability estimate, $\hat{R}$, of the true reliability, R. For a reasonably large number of tests the estimate will be very close to the true probability of success. The point estimate has the desirable statistical characteristic of being an unbiased estimator.

Having an estimate of reliability it might be of interest to know the probability of an exact number of failures in a specific number of trials. The probabilities involved are given by the binomial distribution. If the binomial probability function parameter p is defined as

$$p = 1 - \hat{R}$$

then the parameter is the unreliability or the probability of failure. If is the probability of failure in any given trial and n is the total number of trials, the probability of exactly k failures is given by

$$P(k) = \binom{n}{k} p^k q^{n-k}$$

which is a point estimate of unreliability.

Any estimate of reliability from a portion or sample representative of the population provides a measure from which inferences about the total population are to be made. Therefore, it is more meaningful to express the estimate in terms of limits with an associated confidence rather than expressing reliability as a point estimate.

Confidence limits

Confidence limits are the limiting values of a confidence interval which can be constructed for the true population parameter, say R, and with a specified confidence a statement can be made that R lies within the interval. The confidence associated with this interval is a probability statement expressing the proportion of the time the statement will be correct if intervals are constructed for each of many samples. This proportion is called the confidence coefficient which is selected in advance and generally chosen as 0.90 or 0.95 for reliability estimates.

The interval can be open by considering either the upper or lower confidence limit singly. This one-sided estimate of a confidence limit is frequently employed in reliability statements. A confidence limit is important in reliability statements such as: the reliability, R is at least equal to some specified lower limit or the probability of failure, p, is at most equal to some specified upper limit.

Tables have been generated that provide confidence intervals for the true reliability based on the sample size and the number of reliable items in the sample by Vanderbeck, Dinsmore, and Runchey (17). These tables are constructed for sample sizes from 4 to 400 with confidence coefficients 0.80, 0.90, 0.95 and 0.99. If a reliability test is performed on a pre-selected sample of size N or if the trials are statistically independent, then the results can be used to determine an interval on the true reliability. A statement can be made, that the reliability is within an interval such as

$$R_L \leq R \leq R_u$$

where R_L equals the lower reliability confidence limit and R_u equals the upper reliability confidence limit with an associated confidence level.

Tables of upper confidence limits have been constructed.

The tables present the solution of

$$\sum_{j=o}^{F} \binom{n}{j} \hat{P}u^j (1-\hat{P}u)^{n-j} = 1 - \gamma$$

for Pu where Pu is the upper confidence limit on true probability of failure p, γ is the confidence coefficient, n is the number of trials or sample size, and F is the number of failures observed in the n trials. In the case to be considered, n independent trials are made, and a number of failures F are observed. A statement is then made: "The true probability of failure p is not more than $\hat{P}u$ " (9, p. 1 and 4)

Since reliability can be expressed as

$$R = 1 - p$$

where p is the true probability of failure estimated by the upper confidence limit Pu these tables can be used to determine lower confidence limits for R . If $\hat{P}u$ is substituted for p the expression becomes

$$\hat{R} = 1 - \hat{P}u$$

hence,

$$\hat{R} \leq R$$

which states that $\hat{R}$ is a lower confidence limit for R with confidence coefficient γ .

POISSON DISTRIBUTION

Probability function

The poisson distribution consists of the probabilities of the occurrence of events similar to the binomial distribution. The first event in a similar series is the probability of zero defects or occurrences. Each term of the Poisson expansion corresponds to a term of the binomial expansion and provides the probability of 0, 1, 2, 3, through n defects. Actually this distribution may be derived as a limiting form of the binomial distribution when p is very small but n is so large that np is a finite constant equal to λ . The derivation can be found in Bowker and Lieberman (2), and Roberts (15). The poisson probability function may be written as

$$f(X) = \frac{\lambda^{X} e^{-\lambda}}{X\,!} .$$

The random variable is frequently designated as x, n, r, and k while the parameter is designated as λ and m in some of the literature.

This function provides the probabilities of occurrence of an event, such as a defect. However, there is no finite sample size as there is in the case of the binomial. The event occurs in a defined area of opportunity.

A typical Poisson distribution is shown graphically in Figure 3 for $\lambda = 2$.

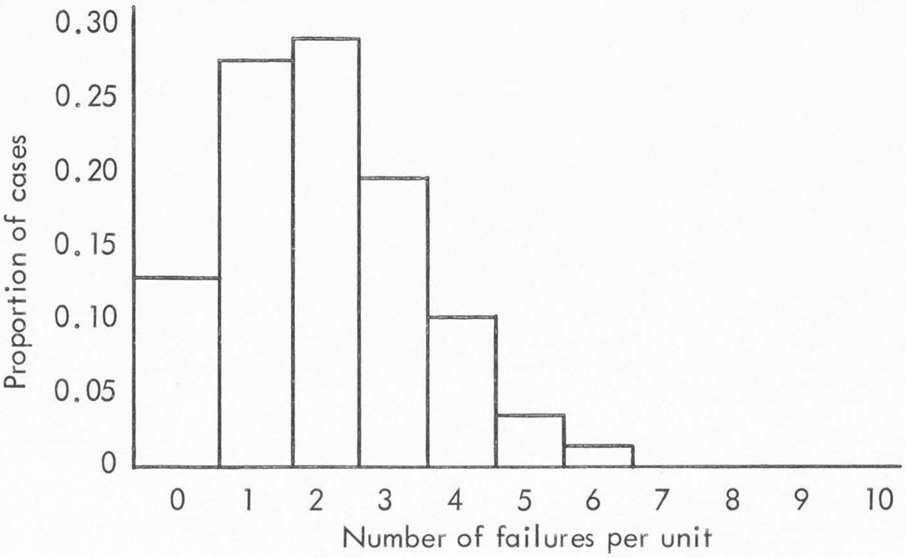

Figure 3. Poisson distribution $\lambda = 2$

Reliability application

The Poisson is the basic distribution that provides the probability of X occurrences in a unit sample or area of opportunity from a source producing an average number of occurrences λ per unit. A unit or area of opportunity may be a sample of a certain number of pieces or it may be a period of time. As an example, consider an assembly operating for ten thousand hours, or ten thousand cycles, or for any particular mission time.

This distribution is a useful approximation to the binomial when n is large and p is very small. This reliability function is also applicable when considering the occurrence of random events in a unit of time. Consider that in a reliability test a failure occurs at a random time, not influenced by the time any other failure occurs, and that the probability that a number of failures will occur in a given time interval is not affected by the number of failures in any other interval of time. Then, if the average number of failures per unit of time is λ , the Poisson distribution function gives the probability of getting X failures in one unit of time. In some cases, the probability of getting X failures in one unit of time may not be as important as the probability of getting at most X failures or of getting at least X successes. Those probabilities are provided by the cumulative Poisson distribution.

The Poisson distribution is also applicable in investigating production defects such as the distribution of the number of defective igniter squibs per lot, where the interest is in determining whether the occurrences are a random result of satisfactory production, or the result of faulty production.

Confidence limits

In establishing a limit on the number of failures that can be tolerated in a test it is necessary to utilize the cumulative probability distribution. This will provide a reliability statement on the probability that there will be some specific number of failures, or less, in an area of opportunity. Tables that give the probability of c or less failures when the expected number is denoted by np' have been developed by Molina (12). In this table c corresponds to x and np' corresponds to λ in the general reliability form of the probability function.

Confidence limits on the parameter λ , which is the average number of occurrences per unit of time or the failure rate per unit of time, can be calculated.

> Suppose the outcome of one observation is n events; e.g., n failures, where n is zero or a positive integer. Then upper confidence limits C on λ, with confidence coefficient γ, can be found by solving
>
> $$\sum_{k=0}^{n} \frac{C^k e^{-C}}{k!} = 1 - \gamma$$
>
> Molina's tables (Ref. 4, Table II) can then be used since in Molina's notation
>
> $$P(c, a) = \sum_{x=c}^{\infty} \frac{a^x e^{-a}}{x!}$$
>
> since $$\sum_{k=0}^{n} \frac{C^k e^{-C}}{k!} = 1 - \sum_{k=n+1}^{\infty} \frac{C^k e^{-C}}{k!} = 1 - \gamma$$
>
> we equate $a = C$ and $c = n+1$ then find the value of a (interpolating as necessary) corresponding to $P(c, a) = \gamma$ in the body of the table. (10, p. 218)

Then a reliability statement can be made that the failure rate is less than or equal to the upper confidence limit C with a degree of confidence equal to γ. This is based on the one observation but is expanded by by the authors to i observations, yielding $n_1, n_2, \cdots n_k$ events respectively.

EXPONENTIAL DISTRIBUTION

Probability function

The exponential probability density function is defined as

$$f(t) = \lambda e^{-\lambda t}$$

where λ is a constant failure rate which completely defines the exponential distribution. A unique characteristic of the exponential density function is that it possesses a time-constant failure rate; that is, the probability of failure of a device whose chance failure pattern follows the exponential is the same for any specified time interval as it is for any other time interval of equal length, given that the device has survived to the beginning of the interval. For example, the probability of failure of such a device in the time interval between 0 and 20 hours of operation is the same for the time interval between 100 and 120 hours, if the device has survived the first 100 hours.

Frequently this density function is expressed as

$$f(t) = \frac{1}{m} e^{-\frac{t}{m}}$$

where m is more commonly known as the mean time between failures. The mean time between failures, abbreviated MTBF, is a time parameter that can be measured directly in hours. However, m is not restricted to being measured in time units. The number of operating cycles may be a more appropriate measure of life than the number of operating hours. In this case m is defined as the mean number of cycles between failures and its reciprocal, λ, is a failure rate per one operating cycle rather than a failure rate per one unit of time.

The exponential is unique since the mean time between failures, m, is equal to the reciprocal of the failure rate. This situation is clarified by Bazovsky (1).

> The exponential density function, like other statistical density functions, also has a characteristic value called the mean. This is obtained for all distributions by forming what is called the first moment $t \cdot f(t)$ of the density function $f(t)$ and integrating the first moment over the entire range of $f(t)$. If we make this operation for the exponential density function we obtain the mean of the exponential function, which we call the mean time between failures:
>
> $$m = \int_{0}^{\infty} t\, f(t)\, dt = \int_{0}^{\infty} t \lambda\, e^{-\lambda t} dt = \frac{1}{\lambda} \quad (1)$$
>
> (1, p. 79)

The mean time between failure is sometimes expressed as, θ possibly since a maximum likelihood estimate of a parameter of any density function is thus designated.

Interpreting the exponential probability density function as a curve from which the probabilities of failure can be obtained by integration, then the probability of failure $Q(t)$, for the interval from zero time to time t is

$$Q(t) = \int_0^t \lambda e^{-\lambda t} dt = 1 - e^{-\lambda t}$$

which is the unreliability. Recalling that reliability is the probability of success and may be expressed as

$$R(t) = 1 - Q(t)$$

then

$$R(t) = 1 - [1 - e^{-\lambda t}] = e^{-\lambda t}$$

for the same period.

A typical exponential distribution is shown graphically in Figure 4 with λ equal to 0.1 failures per hour and a MTBF of 10 hours

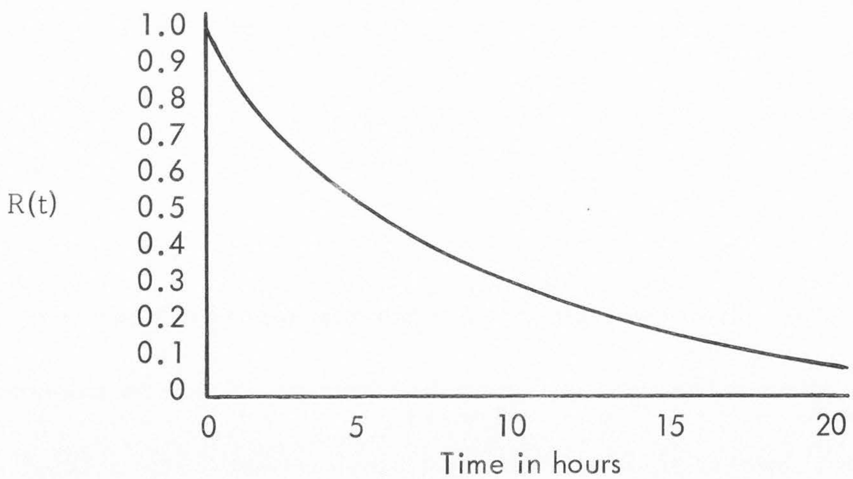

Figure 4. Exponential distribution with $\lambda = 0.1$

An interesting fact to observe concerning the exponential distribution is the probability of observing a failure in less time than the mean time to failure. This probability is computed as

$$\int_{0}^{\frac{1}{\lambda}} \lambda e^{-\lambda t} dt = 1 - e^{-1} = 0.6321$$

where MTBF is expressed as $1/\lambda$. This fact is likely to be contrary to intuition which suggests that the probability would be near one half.

Reliability application

The exponential distribution is characterized by a constant failure rate. Therefore, referring to Figure 1, the life characteristic curve, the constant failure rate is the period between early failures and wearout failures and the only period where the application of the exponential is valid.

In an early paper on life testing Epstein and Sobel (4) proposed a model for life testing which consisted of testing N electronic components or items until n of them had failed, under the assumption that the probability density for length of life is exponential. This model has been extensively applied, and the general consensus is that the exponential distribution is a reasonable assumption in many instances. References are made to a paper by Davis (5) in which failure data on various electronic network components tend to substantiate the hypothesis that life of electronic equipment is exponential. There are many test programs which have been prescribed by the Department of Defence based on the hypothesis of an exponential reliability function.

The error induced by assuming an exponential distribution when in fact the failure pattern follows a Weibull distribution has been investigated by Zelen and Dannemiller (18).

> It is found that these statistical techniques, which are widely used, are very sensitive to departures from initial assumptions. Applying these to life test data when the exponential failure law is not satisfied may result in substantially increasing the probability of accepting components or equipments having poor mean-time-to failure. (18, p. 29)

Although the exponential distribution plays a role in reliability, as the failure characteristics of many components do follow an exponential distribution, some do not, and a careful examination of the validity of the assumption should be made. A number of procedures for testing this assumption are available in a paper by Epstein (6).

> In this paper we give a variety of procedures for testing, on the basis of life test data, whether there are significant departures from an exponential distribution of life. The particular procedures that one should adopt depend on the class of alternatives one is testing against. A number of the tests are based in an essential way on fundamental properties of poisson processes. Questions involving choice of tests are considered, and a number of examples are worked out. (6, p. 83)

Confidence limits

Since the exponential distribution is completely defined by the failure rate, λ, it is completely defined by its reciprocal, m, the mean time between failures. It is necessary to estimate m where the true MTBF is not known and assign confidence limits to this point estimate. In general, when

n devices are originally placed on test, r of them fail at times t_1 $t_2 \cdots, t_r$ counted from the beginning of the test, and the test is truncated at time t_r of the r^{th} failure so that $(n-r)$ devices have not failed at time t_r, then the maximum likelihood estimator for the MTBF is

$$\hat{m} = \frac{\sum_{i=1}^{r} t_i + (n-r)\, t_r}{r} = \frac{T}{r} .$$

This estimator has a probability distribution of its own which is identified to the , χ^2 , chi-square distribution function with $2n$ degrees of freedom. This has been shown by Epstein (16), when the test from which the estimate $\hat{m}$ was obtained was terminated as the r^{th} failure occurred, using the ratio $\frac{2\, n\, \hat{\theta}}{\theta}$ which has the chi-square distribution with $2n$ degrees of freedom, where n is the number of failures resulting from a test.

For a two-sided confidence level $(1 - \alpha)$ we can write the following probability equation, making use of the $\alpha/2$ and $1 - \alpha/2$ percentage points of the chi-square distribution:

$$P\left(\chi^2_{1-\alpha/2;2r} \leq \frac{2r\hat{m}}{m} \leq \chi^2_{\alpha/2;2r}\right) = 1 - \alpha .$$

By rearrangement we can write:

$$P\left(\frac{2r\hat{m}}{\chi^2_{\alpha/2;2r}} \leq m \leq \frac{2r\hat{m}}{\chi^2_{1-\alpha/2;2r}}\right) = 1 - \alpha .$$

Then the two-sided lower confidence limit is

$$L = \frac{2r\hat{m}}{\chi^2_{\alpha/2;2r}} = \frac{2T}{\chi^2_{\alpha/2;2r}}$$

and the upper confidence limit is

$$U = \frac{2r\hat{m}}{\chi^2_{1-\alpha;2r}} = \frac{2T}{\chi^2_{1-\alpha;2r}} .$$

The estimate $\hat{m}$ in the above formulas is defined as

$$\hat{m} = \frac{T}{r} .$$

(1, p. 234)

If it is of interest to know the probability that the true mean time between failure exceeds a specified minimum value then the one-sided lower confidence limit is expressed as

$$C_L = \frac{2r\hat{m}}{\chi^2_{\alpha;2r}} = \frac{2T}{\chi^2_{\alpha;2r}} .$$

The exact lower confidence limit on the reliability R can be expressed as

$$R(\hat{m}) = e^{-\left(\frac{\chi^2_{1-\alpha;2r}}{2rm}\right)t}$$

which says the probability of success is at least equal to $R(\hat{m})$ with $100(1-\alpha)$ confidence.

WEIBULL DISTRIBUTION

Probability function

This three parameter distribution has the unique feature of expressing an increasing, decreasing, or constant failure rate. The probability function is commonly found defined as

$$f(t) = \frac{\beta(t-c)^{\beta-1}}{\alpha} \cdot e^{-\frac{(t-c)^{\beta}}{\alpha}}$$

where in this case it is represented as a function of time (t). In the above equation, α, β and c are parameters of the distribution generally named as follows:

α = Scale parameter

β = Shape parameter

c = Location parameter

The parameter α is analogous to the variance of the normal distribution in that its value affects the scaling of the distribution. The parameter β is termed the shape parameter since its affects the basic shape of the distribution. The parameter c is a location parameter as is the mean of a normal distribution in that it translates the distribution. This parameter c may be interpreted in life length testing as the minimum length of time that passes before any failure can occur.

To study the effect of the shape parameter it is convenient to assume the scale parameter equal to 1. A plot of the Weibull distribution for various values of β with α equal to 1 and c equal to 0 is given in Figure 5.

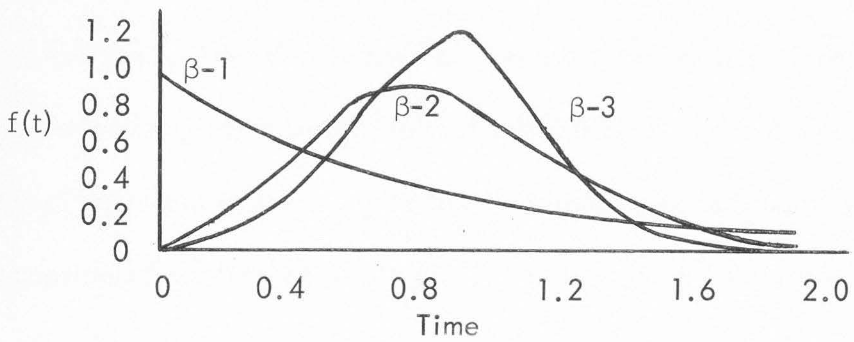

Figure 5. Weibull distribution

It is to be noted that when $\beta = 1$ the Weibull density function reduces to the exponential density function.

The Weibull cumulative distribution function $F(t)$ or the probability of failure $Q(t)$ is expressed as

$$F(t) = \int_{o}^{t} \frac{\beta(t-c)^{\beta-1}}{\alpha} e^{-\frac{(t-c)^{\beta}}{\alpha}} dt$$

$$= 1 - e^{-\frac{(t-c)^{\beta}}{\alpha}}$$

which is the probability of failure in the interval zero time to time t .

Define reliability $R(t)$ as the probability that a system will perform satisfactorily or the system does not fail before time t then

$$R(t) = 1 - F(t) = e^{-\frac{(t-c)^{\beta}}{\alpha}} .$$

<u>Reliability application</u>

This distribution was originally used by Weibull as a probabilistic characterization for breaking strengths. Recently, it has been used by statisticians and engineers in the field of reliability as a model for certain mechanical parts and in the study of life distributions of ball bearings. It is also considered applicable as the distribution of life for some electronic equipment. The life length of many devices can be suitably modeled by determining each of the three parameters. One advantage of using the Weibull distribution is to be able to model so many different life distributions. This distribution can represent exponential, approximately normal, etc.

When the Weibull distribution is assumed as a statistical model the problem of estimating three parameters arises. Two methods, maximum likelihood and minimum chi-square methods are generally applied to estimation problems. Frequently, the location parameter c is assumed equal to zero. This assumption seems reasonable since failures can occur as soon as the experiment is started. The estimation problem then reduces to that of estimating two parameters.

The application of the Weibull distribution of life for some electronic equipment seems justified from the results cited by Roberts (15).

> On the basis of data obtained from failures of more than 2000 electron tubes of various types, a valve of m (shape parameter) was estimated by Kao. This value turned out to be approximately 1.7. Thus $m \cong 1.7$ for electron tubes. With $m = 1$, the Weibull distribution becomes the exponential distribution, as noted above. Therefore, the empirical determination of the value $m = 1.7$ for electron tubes may be considered as evidence that the distribution of times to failure for electron tubes is not exponential. (15, p. 268)

In the above example, m was used to represent the shape parameter in place of β but it represents the possibility of estimating the life characteristics of other types of components from the large amount of data that has been collected throughout industry and government agencies.

Special plotting paper with log log and logarithmic scales known as Weibull probability paper can be used to plot the test data. On such paper the Weibull distribution plots as a straight line of slope β and this plot therefore becomes the basis for deciding that the Weibull is the correct distribution for the given data.

Confidence limits

When the shape parameter of the Weibull density is known, or assumed so that $\beta = \hat{\beta}$ then the exact distribution of the maximum likelihood estimator

$$\hat{\alpha} = \left[\frac{\sum_{1}^{n} (t_i)^{\beta}}{n} \right]^{1/\beta}$$

can be found (6). Confidence limits are established on α and therefor on

$$R = e^{-\frac{T^{\beta}}{\alpha}}$$

which can be expressed as

$$P \left[R \geq e^{-\left(\frac{\frac{T^{\beta}}{\alpha} \chi^2_{2n, 1-\gamma}}{2n} \right)} \right] = \gamma$$

or

$$\hat{R}_L = e^{-\frac{\frac{T^{\beta}}{\alpha} \chi^2_{2n, 1-\gamma}}{2n}}$$

where γ is the confidence coefficient.

This is the lower confidence limit on the probability, R of surviving T hours, or it can be stated that the probability of success is at least this value at the specified confidence.

The general approach to the problem of finding a lower confidence limit on reliability when α and β both are unknown is to find maximum likelihood estimators of the parameters. However, tables have been generated (8) that give the exact lower confidence limits on reliabilities for items whose life times are expressed by a Weibull distribution where both the shape and scale parameters are unknown.

NORMAL DISTRIBUTION

Probability function

A random variable is said to be normally distributed with mean μ and standard deviation σ when its probability density function is

$$f(X) = \frac{1}{\sigma\sqrt{2\pi}} e - \frac{(x-\mu)^2}{2\sigma^2} \qquad -\infty < x < \infty .$$

The random variable is X and μ and σ are the parameters. In terms of descriptives, μ might be considered a location parameter and σ a scale parameter. The normal density does not, unfortunately, lend itself to a simple closed expression when integrated. The transformation

$$Z = \frac{x - \mu}{\sigma}$$

however, brings all normal densities to the same form called the standard or normalized form,

$$f(X) = \frac{1}{\sqrt{2\pi}} e - \frac{Z^2}{2} \qquad -\infty < Z < \infty .$$

Extensive tables of the cumulative standard normal distribution are available.

A graphical representation of the normal probability function is presented in Figure 6.

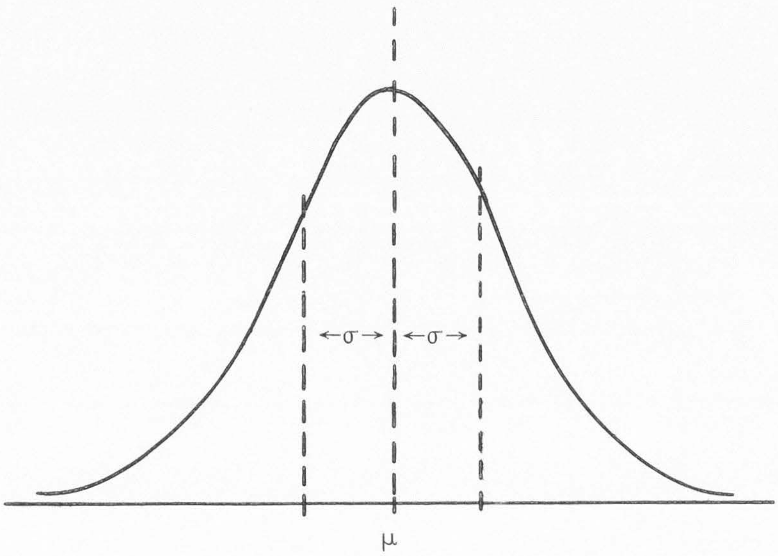

Figure 6. Normal distribution

Reliability application

The probabilities associated with the areas under the normal curve are used to estimate the probability of survival of equipment after it has entered the wearout period of the life characteristic curve. In order to derive probabilities of survival it is necessary to know, M, the mean wear-out life, or have an estimate of it, and also know σ the standard deviation of the lifetimes from the mean

From a large population in which the occurrence of failures is known to be normally distributed, a life test is run on a random sample of size n

until n failures have occurred. Therefore, n measurements of the times to wearout failure are available to compute the mean life as

$$M = \frac{\sum t_i}{n}$$

and the standard deviation as

$$\sigma = \sqrt{\sum \frac{(t_i - M)^2}{n-1}} \; .$$

These estimated parameters can be used in the revised density function

$$f(T) = \frac{1}{\sigma\sqrt{2\pi}} \; e^{-\frac{(T-M)^2}{2\sigma^2}}$$

where M is the mean wearout life, σ is the standard deviation of the lifetimes from the mean M and T is the age or accumulated operating time since new. The cumulative probability distribution or the unreliability is

$$Q(T) = 1 - \frac{1}{\sigma\sqrt{2\pi}} \int_T^{\infty} e^{-\frac{(T-M)^2}{2\sigma^2}} \, dt$$

and reliability is

$$R(T) = 1 - Q(T) \; .$$

The normal distribution is a very good approximation of failures due to wearout, where wearout is defined as the slow abrasion of material, fatigue deterioration of structure, or chemical changes which cause a device to fail to operate.

Confidence limits

A method for obtaining the lower confidence limit on R in the case of the normal distribution is presented by Loyd and Lipow (10).

> We now turn to the problem of estimation of probabilities that a normally distributed variable is contained between two specification limits; i.e., we wish to place confidence limits on the reliability function
>
> $$R = \Phi\left(\frac{T_2 - \mu}{\sigma}\right) - \Phi\left(\frac{T_1 - \mu}{\sigma}\right) .$$
>
> It can be shown using the methods of sec. 8.4 that the following set of formulas can be derived to obtain a lower confidence limit on R_1. Thus, given
>
> $$R = \Phi\left(\frac{T_2 - \bar{x}}{\sigma}\right) - \Phi\left(\frac{T_1 - \bar{x}}{\sigma}\right)$$
>
> one can derive the expression
>
> $$\mathrm{Var}\,\hat{R} = \frac{1}{2}\left[\phi_1^2\left(1 - \frac{1}{2}\Delta_1^2\right) + \phi_2^2\left(1 + \frac{1}{2}\Delta_2^2\right) - 2\phi_1\phi_2\left(1 + \frac{1}{2}\Delta_1\Delta_2\right)\right]$$
>
> where
>
> $$\left.\begin{aligned} \phi_i &= (2\pi)^{-\frac{1}{2}} \exp\left[-\frac{1}{2}\left(\frac{T_i - \mu}{\sigma}\right)^2\right] \\ \Delta_i &= \frac{T_i - \mu}{\sigma} \end{aligned}\right\} \quad i = 1, 2 .$$

Therefore,

$$\hat{R}_L = \hat{R} - K_{1-\gamma} V^{\frac{1}{2}}$$

Is a lower confidence limit on R with confidence coefficient γ where $K_{1-\gamma}$ is the standard normal deviate exceeded with probability $1-\gamma$, and $V^{\frac{1}{2}} \equiv \langle VAR\ \hat{R} \rangle^{\frac{1}{2}}$ where $\langle VAR\ \hat{R} \rangle$ denotes the value of $VAR\ \hat{R}$ when μ and σ are replaced by the estimators $\bar{X}$ and s respectively. (10, p. 205)

An interesting application to solid propellant missile performance parameters is given in an example. In this case they have chosen thrust as the defining parameter and defined reliability as the probability that the thrust will be within some specified requirements. Then the lower confidence limit on the reliability is calculated at a specified confidence.

SUMMARY AND CONCLUSIONS

Summary

The complexity of modern equipment and the demand for more stringent performance requirements have resulted in the need for reliability prediction techniques. Reliability prediction is a necessary part of the design of any physical device; to ascertain the probability that the device will perform its intended function, and to minimize total cost over the required life span of the device.

There is no unique approach to reliability prediction. There are reliability techniques applicable in evaluating data from equipment that is monitored continuously or checked at discrete points in time. The techniques for continuously monitored equipment are derived from continuous density functions and those used in evaluating discrete data are derived from discrete density functions.

Five probability distributions commonly used for reliability estimation were reviewed: The Binomial, Poisson, Exponential, Weibull and Normal. Reliability estimates may be expressed within an interval of values with an associated confidence that the true reliability is within that interval.

Conclusions

The selection of the distribution which best describes the life characteristic of a device is one of the most difficult problems facing the reliability engineer. The validity of the assumption that test data follows a specific distribution should be verified.

If the test results are examined critically, questions arise concerning the best estimate of reliability, confidence limits, and associated confidence coefficient. The statistician can help the reliability engineer obtain answers to these questions based on related statistical techniques.

The objective of this review of some reliability techniques was to generate an interest in the pursuit of the extensive literature and technical material available in the field of reliability.

An interesting project would be to review the literature on the techniques involved in evaluating system reliability when the life characteristics of the subsystems are known to follow different distributions.

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